Probability Theory
Unit 4

Addition Rules

and

Multiplication Rules

Probability of *A* or *B*

- Let *A* and *B* represent events in the same sample space. If *A* and *B* are mutually exclusive events, then

$$P(A \text{ or } B) = P(A) + P(B).$$

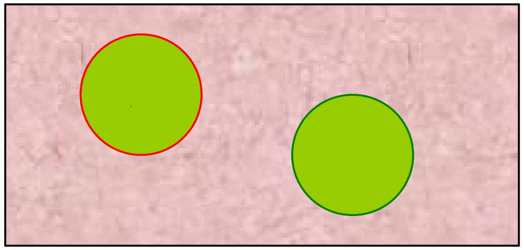
- If *A* and *B* are inclusive events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

Mutually Exclusive

- For mutually exclusive events A and B, the probability of A or B occurring is

$$P(A \text{ or } B) = P(A) + P(B)$$



Mutually exclusive events

- We will look at a mutually exclusive event by using the numbers on one die {1,2,3,4,5,6}.
- Let *A* represent an even number {2,4,6}. *P(A)* = 3/6
- Let *B* represent 3. *P(B)* = 1/6

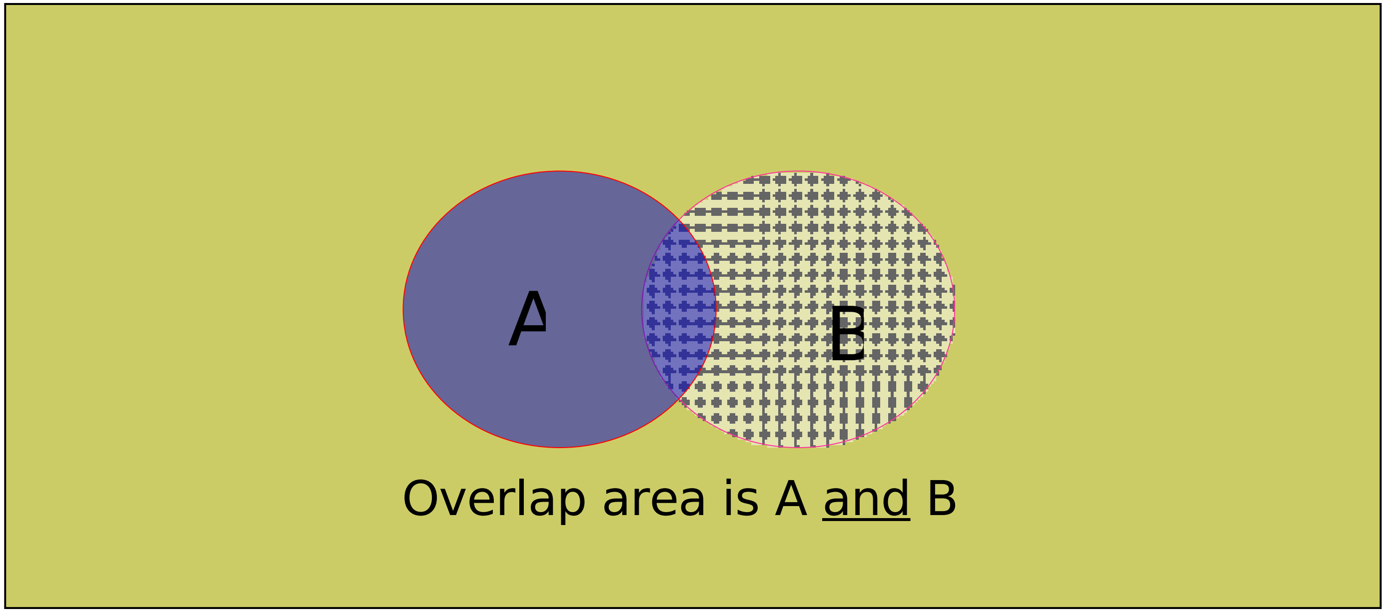
Because *A* and *B* are mutually exclusive you can add *P(A)* and *P(B)* to find *P(A or B).*

- *P*(an even number *or* 3).

  P(A or B) = (3/6) + (1/6) = (4/6), or (2/3).

- In general for events A and B the probability of A or B occurring is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Inclusive events

- We will look at an inclusive event by using the numbers on one die {1,2,3,4,5,6}.

- Let *A* represent an even number {2,4,6}. *P(A)* = 3/6

- Let *C* represent 4. *P(C)* = 1/6

Because *A* and *C* are inclusive you must *subtract P(A and C)* from the sum of *P(A)* and *P(C)* to find *P(A or C).*

- *P*(an even number *or* 4).

  P(A or C) = (3/6) + (1/6) - (1/6) = (3/6),

or (1/2).

Adding Probability

- If $P(A\ and\ B) = 0$,

  Then A and B are disjoint.

Example:

What is the probability of rolling a 4 or a 5?

What is the probability of being dealt a Jack or a heart?

General Addition Rules…

- The union of any collection of events is the event that at least one of the collection occurs.
- <u>At least one</u> means one or both occur.

...for Disjoint Events

If events A, B, and C are disjoint in the sense that no two have any outcomes in common, then

$$P(\text{one or more of } A,B,C)=P(A)+P(B)+P(C)$$

This rule extends to any number of disjoint events.

The addition rule holds if A and B are disjoint but not otherwise.

...for Unions of Two Events

- For any two events A and B,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- If A and B are disjoint, the event that both {A and B} occur has no outcomes in it.

Multiplication Rules

If two events are independent….

What is independence?

Independence is when knowing whether one event occurs does not alter the possibility that the other event occurs.

Independent Events

- If A and B are independent the probability of both A and B occurring is

  P(A and B) = P(A) * P(B)

Example

A = first toss of a coin is a head.

B = second toss of a coin is a head.

P(A and B) = P(A) * P(B) = (1/2) * (1/2) = (1/4) = .25

An example without replacement

Example:

A = 1st card dealt is a heart.

B = 2nd card dealt is a heart.

C = 3rd card dealt is a heart.

A, B, and C are not independent. If A occurs, (ie one of thirteen hearts is dealt for the first card) then there is one less heart to be drawn for the event B, (and 1 less card overall).

P(A and B and C) = (13/52) * (12/51) * (11/50)

= .0129

At least / At Most Probabilities

In Yahtzee, after rolling twice you have a pair of 6's. For your last roll you would like to roll the three remaining dice and get at least one 6. What is the probability of doing so?

This boils down to finding 3 probabilities, rolling 3 dice and getting one 6, two 6's or three 6's. We could find each probability and add them together.

Or we can try something more clever.

Compliment

The complement of getting at least one 6 is getting zero 6’s.

So

P(at least one 6) = 1 – P(no 6’s)

= 1 – [(5/6)*(5/6)*(5/6)]

= .4213